W^m^ Jas Santer R.N.
Tu: 23 June 1908.

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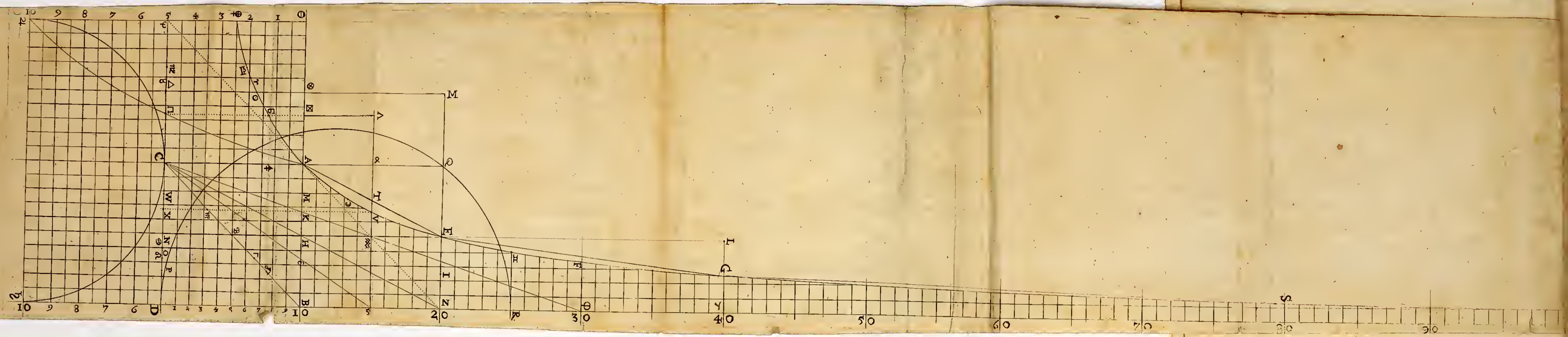
ΓΗΡΑCΚΩ Δ'ΑΙΕΙ ΚΑΙΝΑ ΔΙΔΑCΚΟΜΕΝΟC
Francis Philip Nash.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LOGARITHMOTECHNIA:

OR, THE

Making of Numbers

CALLED

LOGARITHMS

To Twenty five Places,

FROM A

GEOMETRICAL FIGURE,

With Speed, Eaſe and Certainty.

The like not hitherto Publiſhed.

By *EUCLID SPEIDELL*, Philomath.

LONDON,
Printed by *Henry Clark* for the Author, and are to be Sold by *Philip Lea*; at the *Atlas* and *Hercules* in *Cheap-ſide*, near *Friday-ſtreet*, MDCLXXXVIII.

Honours Management; this ensuing Treatise, being the Product of some leisure Hours from that Employment, I hold under your Honours, is with all Duty and Submission, humbly Dedicated, by

Your Honours

Most Faithful and

Obedient Servant,

EUCLID SPEIDELL.

To the READER.

HAving for some Years past shewn to several Persons the Praxis of the following Treatise, and also communicated to them somewhat of the Doctrin leading thereunto, I was often desired not to let them sleep in oblivion, but to publish the same, which was first promoted by my honoured Friend Mr. Peter Hoot Merchant, and seconded by my loving Friend Mr. Reeve Williams, or else they had not seen the Publique; what I have done therein I desire thee to take in good part, being also at proportional charges my self, besides my composing thereof to make it communicable to thee, rather than such an easie and certain way to make Logarithm Numbers (to so many Places) should not be known in our Native Tongue. I have called them Geometrical Logarithms, for that the first Inventors of those Numbers had not adapted Geometrical Figures to them. But the Scheme hereunto annexed having such Properties and Affections as Logarithm Numbers have, hath made me so style them. What I have done herein is to gratifie such who have a Curiosity to examine Logarithm Tables, and to make Logarithm Number to so small Radiuses as are so often printed for common uses with brevity and exactness. Two sheets of the Praxis hereof were printed some time before the rest, which having found kind acceptance with divers, induced me also to let the Remainder be published; and before the printing thereof one was writing upon those two sheets, and was so fair to desire my consent to publish it, which I readily gave, for that I knew him able enough to do it, and when to be at Leisure my self to attend the publishing of the Residue, I knew not. But that not being performed by him, I desire thee to accept of what is done herein as time and leisure hath permitted. I shall not need to write how needful Logarithm Numbers are in those great and useful Arts of Navigation, Astronomy, Dyalling, Fortification and Gunnery, Surveying, Guaging, Interest and Annuities, &c, When, as there are so many Books written and published thereof, not only in our own Language but in many others. And truly the first Inventors thereof are not a little to be had in reverence for making and perfecting those Numbers with so much Labour, as those Methods by which they derived them did require. Here thou mayst make a Logarithm to

7 or 8

7 or 8 places readily and easily; but to 25 places would have been very difficult, if not impossible, for the first Inventors to have produced after their ways. If any thing herein shall offer, whereby thou mayest make farther Improvement, Let the Publique share of the benefit thereof. Thus wishing thee good Success in all thy Studies, is most earnestly desired by,

London,
March, 26. 1688.

E. Speidell.

Advertisement of Characters, *or* Symbols, *used in this Treatise.*

$+$	Signifieth	More, or Addition,
$\times$		Multiplication,
$-$		Less, or Substraction,
$=$		Equal.

Geome-

Geometrical Logarithms.

C H A P I.

ABout the Year $167\frac{5}{6}$, being in Company with *Michael Dairy*, a Citizen of *London* (who had for most part of his life time addicted himself to Mathematical Studies, and hath published divers practical Pieces of several Parts of the Mathematicks, of good Use and Delight) and discoursing about making Hyperbolical Logarithms, I desired him to give me a Rule to make the Hyperbolical Logarithm of 10, from the Consideration of an Hyperbola inscribed within a Right Angled Cone, who gave me this Rule following.

To the Number proposed, *viz.* 10, add an Unite, and substract from it an Unite, and there will be a result of $\frac{9}{11}$: Then divide 1, or 100000000, *&c.* by $\frac{9}{11}$, which is 818181818, *&c.* which Cube *in infinitum* , and divide every one of them (which will be a Rank of Proportional Numbers) by the proper Indices of their respective Powers ; that is to say by 3, 5, 7, 9, 11, *&c.* Then the Addition of all those Quotes will make the Logarithm of 10.

Finding then that 10 divided by $\frac{9}{11}$, maketh 818181818181, *&c.* and to Cube it in *infinitum*, was very difficult, I rejected the Rule, and thought it then not much more easie than *Brigg*'s way : Neither did he tell any Reason or Demonstration for the said Rule ; and because in this Example, I found it so intricate, I did not much care to prosecute it, but neglected it. Not long after he departed this Life ; and since his Death resuming the said thing, and trying if it were serviceable in any other part of the Hyperbola, I soon found it a Jewel, and could make the Hyperbolical Logarithm of 10 at twice, that is to say, from two parts numbered in an Asymptote, whose Fact is 10, with ease, certainty and delight, and have made the Hyperbolical

 Logarithm

Logarithm of 2 to 25 places, in order to see if the learned and laborious *Henry Brigg*'s Logarithms were true to 15 places, which were made after a most laborious and difficult way of Extracting Square Roots, and, as I have heard, was the work of eight Persons a whole Year, and that without any proof, but only if any two or more agreed in their Extractions, Line by Line, Step by Step, it was taken *de bene esse*, which was a work of very great pains and uncertainty: However, they did effect it, and I do find they made the Logarithm of 2 to 15 places very true, as by my Operation, hereafter following, will appear, being done to 25 Places, and afterwards from these Hyperbolical Logarithm deduced *Brigg*'s Logarithms; both which Figurative Operations were performed and examined by me in 8 Hours time. I took this pains to make the Hyperbolical Logarithm to 25 Places; in order also, to see if the most ingenious and laborious *James Gregory*'s Hyperbolical Logarithm would agree with this of mine, which he hath in his *Quadratura Circuli & Hyperbolæ*, Printed at *Padua*; but I find that his Logarithm of 2 corresponds with mine but to 17 Places, I must confess, I did not take the Pains to raise the Logarithm of 2 to 25 Places, according to the Doctrine he hath delivered in that most Learned Piece, but am contented that this easie and certain way I deliver here, and by the Operation thereof the Hyperbolical Logarithm of 2 to 25 Places, is as true in the last as in any where, and may be examined in a few Hours; so that any Body, if he please, may be his own Examiner and Judge, if this Way be not easie, certain and speedy.

Having made several Logarithms for Digit Numbers, and Mixt Numbers, as for $1\frac{1}{4}$, $1\frac{1}{2}$, which are hereafter inserted. The Rule delivered by *Michael Dairy*, is of admirable use and benefit in Squaring the Hyperbola, and making Logarithms from it.

Some time since the death of the said *Michael Dairy*, I shewed unto Mr. *John Collins*, whom I knew had been a great Familiar and Friend to the said *Michael Dairy*, the Figurative Work of my making, the Hyperbolical Logarithms, according to the said *Dairy*'s Rule, who seemed very well pleased with it, acknowledging it to be the speediest way could possibly be, of Squaring the Hyperbola, and making the Logarithms from it, and after a little pausing upon it, replyed, That *Dairy* must

have

have this Rule out of the ſaid *James Gregory*'s Works. I made anſwer, Not from his ſaid *Quadratura Circuli & Hyperbolæ:* He anſwered, No, from his *Exercitationes Geometricæ*, printed at *London*, 1668, a Book I had not ſeen nor heard of till then: And as the ſaid Mr. *Collins* had been always very frank and free to communicate any Mathematical thing to me, ſo I held my ſelf obliged to acquaint him firſt with this Work. He ſeemed to admire, That *Michael Dairy* ſhould keep ſuch a thing from him, who had been ſo great a Familiar with him in theſe Studies. Not long after my diſcovery hereof to the ſaid Mr. *Collins*, he alſo departed this Life; whoſe Death, all that were Mathematical, and knew him, lamented not a little: For he was not only excellent in Mathematical Arts and Sciences, but of a very good, affable and frank Nature to Communicate any thing he knew to any Lover and Enquirer of thoſe things, and hath left behind him thoſe Mathematical Works which will continue his Fame amongſt the Lovers and Students therein. He alſo in his Life time, promoted the Publiſhing of other Men's Mathematical Works; as the Elaborate *Algebra* of the Learned *John Kerſy*, who was my Father's Diſciple about 1645: And alſo of the Learned *Baker*'s Algebra, and ſeveral others. He was a Man of great correſpondence with Mathematical Perſons in foreign Parts, and thereby could give Information of any New or Old Mathematical Book; and till my Acquaintance with him, I was ignorant of Foreign Authors; being but young when my Father dyed, and not then having taken any Pains in theſe Studies: So that by the ſaid *Collins*'s Information and Means, I have heard of, and ſeen, ſome Foreign Mathematical Authors of Note and Eſteem.

After the ſaid Mr. *Collins* had told me of *James Gregory*'s ſaid *Exercitationes Geometricæ*, ſold by *Moſes Pitts* in *Paul*'s Church-Yard, I bought there one of them; and do find that *Michael Dairy* had deduced this Rule from the ſaid Book: Wherein the ſaid *James Gregory* hath made the Squaring of the Hyperbola, an Exerciſe Geometrically demonſtrating the Quadrature of the Hyperbola, ſome time before publiſhed by the Induſtrious and Lucky *Nicholas Mercator*; who by the happy diſcovery of ſome Properties in the Hyperbola, hath made all the Ways of Squaring the Hyperbola flowing from the ſame, very eaſie, certain and delightful: And becauſe neither of them have exemplified

 their

their Doctrine and Rules with Figurative Work, ſo large as to 25 Places. I have here, to Illuſtrate their Admirable Works, inſerted divers Figurative Operations, whereby the Reader and Student may ſee, and have that Satisfaction in Fact and Operation, which is ſo pleaſing and deſirable by every one.

I ſhall not here trouble the Reader with any Sections of the Cone, whereby he may ſee the riſe and geniture of an Hyperbola, from that Body, but content my ſelf to ſhew him from a Square and an infinite company of Oblongs on a Superficies, each Equal to that Square, how a Curve is begotten which ſhall have the ſame properties and affections of an Hyperbola inſcribed within a Right Angled Cone: And ſeeing a Curve made after this manner following, doth become ſuch an Hyperbola, the Doctrines and Analogies delivered and diſcovered by thoſe two Ingenious Artiſts *Mercator* and *Gregory*, may be applyed to this Curve ſo often as need and occaſion doth require.

And not to detain the Reader any longer from knowing how to make this Curve, we proceed to deſcribe the ſame accordingly.

There is a Square ABCD, whoſe Side or Root is 10, let DB be prolonged in *infinitum*, and continually divided equally by the Root, or DB, and thoſe Equal Diviſions numbered by 10, 20, 30, 40, 50, 60, 70, *&c.* in *infinitum*: Upon theſe Numbers let Perpendiculars be erected, which call Ordinates, and each of thoſe Perpendiculars of that length, that Perpendiculars let fall from the aforeſaid Perpendiculars to the Side or Baſe CD (which call Complement Ordinates) the Oblongs made of the Ordinate Perpendiculars, and Complement Ordinate Perpendiculars may be ever Equal to the Square AD, which is eaſily done thus, for it is $\frac{100}{20}$ $\frac{100}{30}$ $\frac{100}{40}$ $\frac{100}{50}$ *&c.* produces the Length of the Ordinate Perpendiculars; for 100 divided by 20 maketh 5 for the Length of the Ordinate Perpendicular 20 E. And 100 divided by 30, giveth 3333333, *&c.* for the Ordinate Perpendicular 30 F. And 100 divided by 40 produceth 2·5 for the Ordinate 40 G, and ſo of the reſt. And Geometrically it is as 20 D is to BD, ſo is AB to AH, equal to 20 E, as before, for that the Angle ACH is equal to C 20 D, and ſo of the reſt. And for the Length of the next Ordinate, you ſay, as 30 D to BD; ſo AB to AK, which is alſo Equal to 30 F. And for the Ordinate 40 G, ſay, as 40 G to BD, ſo AB to AM, which will be Equal

equal to 40 G, and ſo of all the reſt, whereby you have all the Perpendiculars upon the prolonged SideDB,both Geometrically and Arithmetically ; the ſame Proportion is to be obſerved for any Intermediate Parts.

Now, for all the Perpendiculars which are let fall from the aforeſaid Perpendiculars or Ordinates to the Baſe CD, which call Complement Ordinates, the Geometrical Proportion for NE,equal D 20 is as HA to AC,ſo CD to C 20 equal to NE,and for the Complement Ordinate OF equal D 30, it is as KA to AC, ſo CD to D 30 equal OF, and ſo of the reſt. Now for NE Arithmetically, ſay, as 5 to 10, ſo 10 to 20 equal to NE, equal to D 20 ; and for OF, ſay, as 33333333 to 10, ſo 10 to 30 equal OF, which is equal to L 30, and for PG equal to D 40, ſay, as 25 is to 10, ſo 10 to 40 equal to PG, equal to D 40; and ſo for all the reſt of the Complement Ordinates ſtanding upon the Baſe CD, whereby it doth appear, That all the Oblongs made of the Ordinates, and Complement Ordinates are each of them equal to the Square AD, which is here 100 ; for the Oblong ED being made of E 20 and D 20, is by the 13 of of the 6 Euclide equal to the Square AD, for Q 20 is a Mean Proportional between D20, and 20 R, and Q 20 is found to be equal AB, ſo is the Oblong or Parallelogram ED equal to the Square AD, and the like Demonſtration ſerves for all the Oblongs or Parallelograms ſtanding upon the Baſe CD, by the Tips or Angular Points of thoſe Parallelograms,or from the Ends of all the Ordinates ſtanding upon 20, 30, 40, 50, 60, 70, *in infinitum*;draw the Curve Line from A towards E,ſo ſhall you deſcribe the Curve AEFGS, which Curve you ſee is begotten without any conſideration or reſpect to the Section of a Cone, and yet becomes the ſame in all reſpects, to have the ſame Affections and Properties of an Hyperbola derived from the interſecting of a Right Angled Cone, as ſhall be ſhewed in the next Chapter.

You may obſerve the Complement Ordinate NE, being equal to D 20, is equal to twice Radius. And if CD be made the Radius of a Circle, then is NE equal to D 20, equal to the Tangent of twice Radius, for D 20 becometh the Tangent of twice Radius. Alſo it is manifeſt that the Complement of the Tangent equal to twice Radius, is alſo equal to half the Radius ; That is, the Tangent Complement of D 20, is 20 E equal to 5.

And

And seeing the Radius is ever a mean Proportion between the Tangent and the Tangent Complement, therefore each Oblong is equal to the Square AD.

CHAP. II.

IN the former Chapter, we have shewed the begetting of a Curve, without any regard to the Section of any Solid Body; and now it remaineth to prove that this Curve hath the same Properties and Affections that an Hyperbola, deduced from the Section of a Right Angled Cone.

I remember some time before the death of *John Collins*, he told me, It was a great Work of the Learned *Vincent* or *Magnan*, to prove that Distances reckoned in the Asymptote of an Hyperbola, in a Geometrical Progression, and the Spaces that the Perpendiculars, thereon erected, made in the Hyperbola, were equal the one to the other. This Property is now very well known, the Hyperbola hath, and this Curve hath the same Property; which is discernable almost *intuitu*. In the Hyperbola, they call the Prolonged Line DB *in infinitum*, from the Point B, an Asymptote. And here in this prolonged Line from B, on 20, 40, 80, 160, 320, 640, *&c.* let the Ordinates touch the Curve in EFGS, *&c.* I say, That those Trapezias with the Curve Line (or Hyperbolical Spaces) are all equal the one to the other. In the Right Lined Trapezias thereon, it is manifest they are all equal the one to the other by several Propositions of the 6th Book of *Euclid*: For in the Right Lined Trapezia ZEAB the side AB is twice EZ, and by the former Chapter it was found that GY is half EZ, by saying, As AB to EZ, so EZ to GY. And the Right Lined Trapezia ZEAB shall be therefore equal to 75: Now, forasmuch, as in the Right Lined Trapezia YGEZ the Base of that YZ, is double to ZB, but the Perpendiculars are in the *Ratio* of AB to EZ; for as before, it is as AB : EZ :: EZ : GY, therefore the Right Lined Trapezia YGEZ equal to the Right Lined Trapezia ZEAB, and so will all Right Lined Trapezia's, so Based and Perpendicular'd, be equal the one to the other. The Trapezia LYEZ is equal to

to the Square AD, becaufe ZY × ZE is equal to AB × AB, as in the foregoing Chapter, the Oblong GZ is half the Parallelogram LZ, and the Triangle GEI half the Parallelogram LI. Now the Parallelogram GZ + the Triangle GEI (half the Parallelogram LI) is equal to the Right Lined Trapezia ZEAB, for in Numbers 20 × 2,5 = 50 + half 50 equal to 75.

Thus you fee the Right lined Trapezias numbred upon the prolonged fide in a Geometrical Proportion are equal the one to the other; it remaineth now to prove the mixed Trapezias, that is, the Trapezias ftanding upon the fame bafis but joyned aloft with this Curve are alfo equal the one to the other.

Firft let it be obferved, That thefe Curvilined Trapezias (or Hyperbolical Spaces) are ever lefs than the Right lined Trapezias, becaufe all the Points in the Curvilined Trapezias fall within the Right line that joyns the Right lined Trapezias: And is thus proved in the Right Lined Trapezia BZEA; let there be in the Bafe Z B upon the point 5 Erected a Perpendicular to touch E A in T then is T 5 equal to A B lefs α T which is half Q E, that is, 10 lefs 2, 5 equal 7, 5 = T 5. But by the foregoing Chapter, if a Perpendicular be erected upon the faid Point 5 to V, (to touch the Curve in V) fo that the Parallelogram VD fhall be equal to AD, as in the former Chapter, then will it be AD divided by D5 = 5 V, which is 100 divided by 15, produceth *6 666666* for the true Length of 5 V, whereas before 5 T is 7, 5. By the fame means may all the intermediate Points in this Curve Line EVA be found to fall within the Right Line AE, that is, between the Line EA and ZB, and therefore the Right Lined Trapezia ZETAB greater than the Curvilined Trapezia (or Hyperbolical Space) ZEVAB.

Now, forafmuch as we have proved that the aforefaid Right Lined Trapezias are ever equal the one to the other, it will now follow, That feeing the Curve paffing by all thofe Points which are Extremities of the Right Lined Trapezia, (as well as the Curvilined Spaces, being upon the fame Bafes always) and this Curve being generated continually by one and the fame Ratio, as in the former Chapter. That therefore the Curvilined Trapezias ftanding upon Geometrical Proportional Bafes, fhall be alfo equal the one to the other, which is the Affection and Property of the Hyperbola. And fo the Doctrines and

Precepts

Precepts delivered by thoſe two Famous Geometers, *Mercator* and *Gregory*, for the Squaring of the Hyperbola, be applyed to this Geometrical Curvilined Figure ; and from it derived Logarithms, which may be called Hyperbolical Logarithms.

The way and means to find the Hyperbolical Spaces in Numbers, ſhall be ſhewed in the following Chapters.

C H A P. III.

IN this Chapter we will conſider that moſt admirable diſcovery (I ſuppoſe *Mercator* made) upon drawing the Diagonal CB, which by conſtruction cutteth all the Perpendiculars ſtanding upon the Baſe CD at Equal Angles, and in ſuch Diſtances from the Baſe CD, as doth unravel the Myſtery of his infinite Series, and make the Quadrature of the Hyperbola more eaſie and certain than any I ever ſaw or heard of.

The Diagonal CB being drawn doth give the firſt Term of a Geometrical Progreſſion or Infinite Series between 10 and 20, or 30, 40, 50, 60, 70, 80, 90, *&c.*

That is to ſay, Would you know the firſt Term of an Infinite Series (or Numbers Geometrical Proportional continued) between 10 and 20, the ſumm of all which ſhall be juſt 20. Having from Z drawn the Line ZC, to cut BA in H, which taken off, and applyed to CD, from C to N equal NB, becauſe the Angle BCN is equal to the Angle CBN: I ſay that NB is the firſt Term of an Infinite Series between CD equal AC, and the Perpendicular NE equal DZ, which may be done by Squaring AC, and dividing it by the Side or Number given, the Complement whereof to 10 is the firſt Mean or Term of that Infinite Series, ſo ſhall the firſt Term of the Infinite Series between 10 and 20 be found 5 : Thus in Numbers, $10 \times 10 = 100\ \frac{100}{20} = 5$, the Complement whereof to 10 is 5, equal CN, equal NB for the firſt Term of an Infinite Series between 10 and 20, whoſe ſumm

is 20

is 20, as by the Arithmetical work in the Margent where *a*, *b*, *c*, *d*, *e*, *f*, *&c.* are a Rank of Geometrical Progressional Numbers, whose infinite Sum would make but 20, and is demonstrated by the 7 [illegible]

a.	10
b.	5
c.	2.5
d.	1.25
e.	. . 625
f.	. . 3125
h.	. . 15625
i.	. . . 78125
k.	. . . 390625
l.	. . . 1953125
m.	 9765625
	19. 99. 0234375

And in Numbers thus. As 10 less 5 is to 10 what 10? the Quotient will be found 20 for the whole Summ of that infinite Series between 10 and 20 whose first Term is 5

In like manner would you know the first Term of an infinite Series between 10 and 30 Divide the Square of ac = 100 by 30 the Quotient will be found 3333333 whose Complement to 10 is *6666666* I say, That *6666666* is the first Term of an Infinite Series between 10 and 30, as by the Arithmetical Operation in the Margent; And briefly thus, as 10: :3333333=*6666666*: 10 :: 10 : 30 so is 30 the whole Summ of all those infinite Progressional Numbers between 10 and 30. In the figure you draw the line ΦC which cutteth BA in K, I say that BK transferd from C to O equal Or is the first Term of an infinite Series between AC and OF equal DΦ. And PS be the first Term equal 7, 5 equal PC between AC and PG equal DY = 40 between 10 and 40, for as before 10 —7, 5 =2, 5: 10 :: 10 = 40 so is 40 the whole Summ of an infinite Series between 10 and 40, whose first Term is 7, 5 and so of all the rest. On this great Mystery depends much the following squaring of the Hyperbola and hath made it so intelligible and easie.

a.	10.
b.	. 6. 666666666
c.	. 4. 444444444
d.	. 2. 962962962
e.	. 1. 975308641
f.	. 1. 316872427
g.	. . . 877914951
h.	. . . 585276634
i.	. . . 390184422

Hence you may Note, That if you would know the first mean of an infinite Series between any other Number, and that Number doubled, tripled, quadrupled, *&c.* you may from this Root 10 deduce it, as for Example, let CD be 12 and I would know the first Term of an infinite Series between 12 and 24 the double of 12 I say as 10 is to 5 what 12 *facit 6.* for supposing CD = 10 it was before found 5. Therefore between 12

and 24 you say 10 : 5 :: 12 : = 6 for the first mean of an Infinite Geometrical Progression between 12 and 24. And is thus by the former Analogy proved by saying as 12—6=6 : 12 :: 12 := 24 so is 24 found to be the total Summ of an infinite Series between 12 and 24 as by the Operation in the Margent will appear.

a.	12
b.	6
c.	3
d.	1, 5
e.	. . 75
f.	. . 375
g.	. . 1875
h.	. . . 9375
i.	. . 46875
k.	. . 234375
l.	. . 1171875
m.	. . . 5859375
	23, 994140625

Also if it were required to find the first mean between 12 and three times that number *viz*.36. Say as 10: *6666666*: 12:: 8. And so by Tabulating or working this Example as you do the former the total Summ of that infinite Series between 12 and 36 (the first Term being found 8) will amount to 36 : And for proof you say 12 -- 8 = 4: 12 :: 12 : = 36 so is 36 the whole Summ of that infinite Series or Geometrical Progression between 12 and 36, the first Term being 8 as was desired.

If it shall be required to know an infinite Series between 10 and any other Number, as to know the first Term between 10 and 15 that is, between DC and D5, draw 5C, and it cutteth BA in E, I say that BE is the first Term of an infinite Series between 10 and 15 as by the Operation in the Margent, and as before taught $\frac{100}{15}$ = *6666666*, which substract from 10 leaveth 3333333 for the first Term. And by the former Rule is proved thus ; As 10--3333333=*6666666* : 10 :: 10 : =15. Thus may you find the first of any Term of an infinite Series between 10 and any other Number.

a.	10
b.	3333333333
c.	1111111111
d.	370370370
e.	123456793
f. .	41152264
g. .	13717421
h. . .	4572473
i. . .	1524157
k. . .	508052
l. . .	169350
m. . . .	56450
	14, 999971774

And if it should be desired to know the first Term of an infinite Series between any two other Numbers. As for Example, I would know the first Term of an infinite Series between 12 and 16, to do it Geometrically you must suppose BA = 12. And then counting 16 from D in the line DB prolonged by that Point, and C draw a line which will cut the line.

line BA (now repreſenting 12) in a point, which taken from B, will be the length of that line Geometrically, and Arithmetically it will be found by ſaying, as 16 D to DC, ſo 16 B = 4 to B 3 in the line BA numbred from B to A when BA is 12. In Numbers the Proportion ſtands thus, 16 : 12 :: 4 : = 3 which 3 is the firſt Term of an infinite Series between 12 and 16 as was required, and is manifeſt by the Operation in the Margent, which to prove by the foregoing Rules you ſay, as 12—3=9 : 12 :: 12 : =16; which is to ſay, as 9 to 12, what 12 *facit* 16 for the whole Summ of that infinite Series between 12 and 16, the firſt Term being found as before to be 3.

a.	12
b.	.3
c.	. .75
d.	. .1875
e.	. . .46875
f.	. . .1171875
g.	29296875
h.	732421875
i.	 18310546875
	15, 99993896484375

Thus have you that great Myſtery unfolded of finding Geometrically and Arithmetrically the firſt Term of a Geometical Progreſſion with the whole Summ of that infinite Progreſſion or Series between any two Numbers, which is the main thing I conceive that famous *Mercator* was ſo lucky in diſcovery thereof, and doth unravel the Myſtery of ſquaring the *Hyperbola*, as will be manifeſt in the next Chapter following.

CHAP. IV.

I Obſerve from the ſaid Learned *Gregory's* *Exercitationes Geometricæ*, he giveth three Quantities or Spaces contiguous to the *Vertex* A, which ſhall be all equal the one to the other which is very true and perſpicuous, and then ſhews how to find the *Areas* of them ſeverally as in page 9, 10, 11, and 12 of ſaid Book.

And here we muſt conſider them all three before we come to underſtand *Dairy's* Rule, which is but a Deduction from theſe, as will appear hereafter. And now I begin to conſider the ſaid three ſeveral Quantities or Spaces all contiguous to the

the *Vertex* A, and of a different form, and yet equal the one to the other.

Let it therefore now be ſhewn thoſe three ſeveral Spaces differing in form, and yet equal the one to the other contiguous to the *Vertex* A, which ſhall repreſent the Curvilined *Trapezia*, or Hyperbolical Space for 2. The firſt Curvilined *Trapezia* or Hyperbolical Space for 2 let be ZBAVE, which is intelligible *intuitu*. The ſecond let be AVENC, which is equal to the former ZBAVE by the 43 of the 1 of *Euclid*, becauſe the Parallelogram EZBH (BH being equal to ZE) is equal to the Parallelogram HACN (HA being equal to HB.) Now for as much as the Curvilined Triangle AVEH is common to both the ſaid Curvilined *Trapezias* or Hyperbolical Spaces, it remaineth therefore, that theſe two Curvilined *Trapezias* or Hyperbolical Spaces ZEVAB, and AVENC are equal the one to the other. And now to find out the third Curvilined Space contiguous to the *Vertex* A, and yet equal to either of the other two, but differing in form, doth require a little further conſideration which from him is directed thus. And is manifeſt by the figure, divide BZ in two equal parts in 5. Then as before taught will it be as 5 D : DC : : 5 B : BK=DX make CX equal to CΠ, or to find DΠ it is as D 5 : DZ : : DC : DΠ upon Π erect a Perpendicular to touch the Return or Continuation of the Curve on the other ſide of A, from A towards Θ in Σ. So is this Curvilined Figure or Hyperbolical Space ΠΣAVX (differing in form) equal to either of the other two ZEVAB or CAVEN. And from finding the *Area* of this Curvilined Figure or Hyperbolical Space ΠΣAVX is derived *Dairy*'s Rule, which is but a Deduction from the finding of the *Areas* of the other two, as will hereafter appear.

Arithmetically DX is found by ſaying, as 15 to 10 what 10 ? *facit* *66666666*, and 10 leſs *66666666* reſt 33333333 for CΠ ſo is XΠ equal to *66666666*, or DΠ may be found thus ; as D5 : DZ : : DC : DΠ, which in Numbers is, As 15 . 20 : : 10 : = 1. 33333333 for DΠ.

James Gregory in the 4 Propoſition page 10, 11 of his *Exercitationes Geometricæ* doth contemplate firſt, the Second of theſe three Curvilined *Trapezia* or Hyperbolical Spaces, that is to ſay, the Curvilined *Trapezia* or Hyperbolical Space CAVEN, and in that 4 Propoſition after and long a learned Demonſtration

	The Infinite Series of Numbers Proportional.			*The Quotes to be Added.*	
A	50,0000000000000	I	*	50,0000000000000	A
B	25	II	--	125	B
C	125	III	*	41666666666666	C
D	625	IV	—	15625	D
E	3125	V	*	625	E
F	15625	VI	—	2604166666666	F
G	78125	VII	*	1116071428571	G
H	390625	VIII	—	48828125	H
I	1953125	IX	*	217013888888	I
K	9765625	X	—	9765625	K
L	48828125	XI	*	44389204545	L
M	244140625	XII	—	20345052083	M
N	1220703125	XIII	*	9390014037	N
O	6103515625	XIV	—	4359654017	O
P	30517578125	XV	*	2034505206	P
Q	15258789062	XVI	—	953674316	Q
R	7629394531	XVII	*	448787914	R
S	3814697265	XVIII	—	211927625	S
T	1907348632	XIX	*	100386770	+
V	953674316	XX	—	47683715	
W	476837158	XXI	*	22706531	
X	238418579	XXII	—	10837208	
Y	119209289	XXIII	*	5183013	
Z	59604644	XXIV	—	2483527	
	29802322	XXV	*	1192093	
	14901161	XXVI	—	573122	
	7450580	XXVII	*	283355	
	3725290	XXVIII	—	133046	
	1862645	XXIX	*	64229	
	931322	XXX	—	31044	
	465661	XXXI	*	1502[illegible]	
	232831	XXXII	—	7276	
	116415	XXXIII	*	3527	
	58207	XXXIV	—	1712	
	29103	XXXV	*	831	
	14551	XXXVI	—	404	
	7275	XXXVII	*	196	
	3637	XXXVIII	—	96	
	1818	XXXIX	*	46	
	909	XL	—	22	
	454	XLI	*	11	
	277	XLII	—	5	
			*	2	
			—	1	

10, 00000000 =
The whole Summ
A—B : A :: A =

6, 66666666 =
A—C : A : : A =
Impares

3, 333333333 =
Pares
B—D : B : : B : =

A--B. C- D. E--F. G--H
= 3, 333333333

A—B+C—D+E—F+
= 3, 333333333

A+B+C+D+E+F+G
A--B+C--D+E--F+G
$=A+\frac{2}{3}+\frac{2}{5}+\frac{2}{7}+\frac{2}{9}+\frac{2}{11}+\frac{2}{13}$

Logarithm of 2. } 693147180559945

Half the Logarithm of 3 } 549306144334055

Logarithm of 3 } 1098612288668110

Logarithm of the differ. between 2 & 3 or the Log. of 1, 5. } 405465108108165

143841036225890

287682072451780

monſtration doth prove the Space CAVEN to be equal to his ſuppoſed quantity ω, and then refers you to *Caveleriu*'s method of Indiviſibles, a Book I have not yet ſeen. Which briefly I conceive may be thus eaſily demonſtrated.

It is manifeſt that all the Perpendiculars let fall from the Ordinates (ſtanding upon the prolonged ſide DB) to the Baſe CD, doth not only deſcribe the Curve, but would alſo fill the whole Hyperbolical Space, were Number, and the Curve Definitive. And thoſe Perpendiculars let fall from the Ordinates (ſtanding upon DB prolonged, numbred 20, 30, 40, 50, 60, *&c.*) doth divide the Baſe DC, in $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$, *&c.* And whereas the Diagonal CB, by croſſing all thoſe Perpendiculars doth give the firſt Term of an infinite Series between the Root or Side AC (= NH) and the length of each of thoſe Perpendiculars ſtanding upon the Baſe CD. Therefore to know the Hyperbolical Space CAVEN, divide the Parallelogram CH, making it the firſt Term of an infinite Series by the *Ratio* of NB = NC to NH *in infinitum*, and each of thoſe Quotes or Proportional Numbers by 1,2,3,4,5,6,7,8, *&c.* alſo *in infinitum.* The Quotes of all the laſt Diviſions added, will give you the *Area* of the Hyperbolical Space CAVEN, and ſo for any other Curvilined or Hyperbolical Space ſtanding upon the Baſe CD, as by the Calculation following.

And before I handle any of the other two Curvilined Spaces differing in form, and yet equal to this Hyperbolical Space CAVEN, we will exemplify this Demonſtration in Operation, and the figurative work thereof, ſhall be the work of the next Chapter.

C H A P. V.

LET it be required to calculate the *Area* of the Curvilined *Trapezia* or Hyperbolical Space CAVEN to 15 Places, and hereafter you will ſee it done to 25 Places according to *Dairy*'s Rule in Chapter VII. but with greater diſpatch.

The

Thus by the Calculation do you find the *Area* of the Curvilined *Trapezia* or Hyperbolical Space CAVEN to the 15 place to be 693147180559945 equal to the Curvilined *Trapezia* ZEVAB, and alſo equal to the Curvilined *Trapezia* or Hyperbolical Space ΠΞAVX, the Calculation of the *Areas* of any part of theſe two latter ſhall be ſhewn hereafter which will differ in Operation, yet bring out the ſame Number, and in Calculating the laſt, we ſhall uſe *Dairy*'s Directions.

It having been before ſhewn that the Hyperbolical Space ZEVAB equal to the Curvilined *Trapezia* or Hyperbolical Space CAVEN is equal to the Curvilined *Trapezia* or Hyperbolical Space YGFEZ, that therefore the ſaid ZEVAB is a Space or Quantity to repreſent the Logarithm of 2. So then the aforeſaid Number 693147180559945 is an Hyperbolical Logarithm of 2. And having the Logarithm of 2. you have alſo the Logarithm of all the Powers of 2.

And by this Calculation you have not only gotten the Logarithm of 2, but gained alſo the Logarithm of 3, for if you add all the Quotes marked with this Aſteriſm (*) the Addition of them ſhall be the half Summ of the Hyperbolical Logarithm of 3. agreeable to the 4*th*. Confectary of the 4*th*. Propoſition, and firſt inference on the 5*th*. Propoſition of ſaid *James Gregorye*'s *Exercitationes Geometricæ*, from whence 'tis plain that *Michael Dairy* had his Rule, as will appear more manifeſt after we have contemplated the two other Curvilined *Trapezias* or Hyperbocal Spaces ZEVAB and ΠΞAVX.

The Addition of the Quotes marked with * make 549306144334055 which doubled is 1098612288668110 for the Logarithm of 3, and now having gotten the Logarithm of 3, you have alſo the Logarithm of all the Powers of 3, and of all the Compoſites of 2 and 3.

Again if you ſhall from 50 ſubſtract 125, and to that add 4166666666666, and from that ſubſtract 15625 and ſo on throughout, you ſhall have the Logarithm of the difference between 2 and 3, or the Logarithm of 1 and ½ or 1 and ,5 correſpondent to the inference on the 5 Prop. of *James Gregory*, all which ſhall be fully exemplifyed hereafter.

The Calculation of the Logarithm of 2, according to the Method before going is the ground work of all the Calculations following, and I ſhall only give the Calculation of one more

Space

Space to repreſent the Logarithm of 3 after that Method, though we have, you ſee, gotten already the Logarithm thereof, but ſuppoſing we had not, and were to find that firſt according to the ſaid Method.

From F you let fall a Perpendicular as FO. So is the Curvilined *Trapezia* or Hyperbolical Space AEFOC equal to the Curvilined *Trapezia* or Hyperbolical Space AEFΦB for the Oblong AEOC as before ſhewn will be equal to the Parallelogram EFΦB, and the Curvilined Triangle AEFE common to both Parallelograms. Therefore the Curvilined *Trapezia* AEFOC equal to the Curvilined *Trapezia* or Hyperbolical Space AEFΦB, to calculate whoſe *Area* which will be the Logarithm of 3 you proceed as followeth, which Work is but partly done, to ſhew the way thereof, the Logarithm of three being hereafter done to 25 places; but with far greater diſpatch than this Method will permit.

6666666666666666	:	I	6666666666666666
4444444444444444	:	II	2222222222222223
2962962962962962 96	:	III	.98765432098765
197530864197530	:	IV	.493827160493 82
131687242798352	:	V	.263374485596 71
87791495198902	:	VI	.146319158664 84
58527663465934	:	VII	..836109478084 8
39018443310630	:	VIII	..487730528882 9
26012294873752	:	IX	..289025498597 3
17341529915834	:	X	..173415299158 3
11561019943888	:	XI	..105100181308 1
7707346629258	:	XII	...642278885771

I have but gone twelve ſteps in the Calculation of the Logarithm of 3 after ſaid Method, which will if it were added but give the Logarithm of 3 to five places. I have left it unfiniſhed for the Exerciſe of thoſe who ſhall take delight herein, and finiſh it throughout to the intent of making the Logarithm of 3 to fifteen places according to this Method. By adding the Quotes of ſo much as is done the firſt five figures will be 10986 correſpondent to the Logarithm of 3; this Method being ſomewhat ſlow, I ſhall not Calculate the Logarithms of any other Numbers according to it. And by theſe two Examples the Reader may ſee enough to calculate any other Curvilined *Tra-*

pezia

pezia or Hyperbolical Space ſtanding onely in or upon the Baſe CD equal to any Hyperbolical Space: reckoned in the prolonged ſide DB.

And ſo we will contemplate in the next Chapter the Affections and Properties of the Hyperbolical Space or Curvilined *Trapezia* AΣΘYC equal to the Curvilined *Trapezia* or Hyperbolical Space ABZEV.

CHAP. VI.

BEfore we ſhew how to calculate any part of the Curvilined *Trapezia* or Hyperbolical Space AΣΘΥC, equal to the Curvilined *Trapezia* or Hyperbolical Space AVEZB. We will inſert Tables to illuſtrate the 1, 2 and 3 Propoſitions of *James Gregory*'s *Exercitationes Geometricæ*.

The

The First Table of Illustration.

I — II

I	II
A	5
B	25
C	125
D	625
E	3125
F	15625
G	78125
H	390625
I	1953125
K	9765625
L	48828125

Z = 9999999999

The Rule to find Z.

5--25=25:5 :: 5 : =10=Z.

III

A C E G I L

The Rule to find Z is

5--325=325:5 :: 5 : =

66666666 Impares = Z

Z =

IV

B D F H K M

The Proportion to find Z

25--625=1875:25 ::25:=

33333333 Pares = Z

Z =

V

A-B C-D E-F G-H I-K L-M

The Rule is

25-625=1875:25::

25 : = 33333333

Equalis Paribus

VI

+A –B +C –D +E –F +G –H +I –K +L –M +N

The Proportion is

5+25=75 : 5 :: 5 : = 33333333

Equalis Excessui Imparium supra omnes Pares.

VII

A +B –C +D –E +F –G +H –I +K –L +M –N

The Proportion is

5-125=375: 5 :: 5 : =66666666

Equalis Paribus

VIII

+A+A
+B–B
+C+C
+D–D
+E+E
+F–F
+G+G
+H–H
+I+I
+K–K
+L+L
+M–M
+N+N

2A+ 2C+ 2E+ 2G+ 2I+

The Explanation of the foregoing Table.

This Table conſiſteth of Eight Columns, The firſt is a ſuppoſed literal Rank of Quantities continually proportional. The ſecond is of Numbers correſpondent to the firſt in a Ratio, as 2 is to 1. Or 5 to 2,5. What 2,5 ? And ſo ſuppoſed continued *in infinitum*, with the Rule how to find out the whole Summ of thoſe Numbers ſo continually proportional.

The third Column ſheweth how to find the whole Summ of the odd Quantities or Numbers.

The fourth teacheth how to know the whole Summ of the even Quantities or Numbers.

The fifth telleth how to find the whole Summ of the Difference of the Quantities of the firſt Column.

The ſixth ſuppoſeth A--B-|-C--D-|-E, and teacheth how to find the Solution thereof.

The ſeventh ſuppoſeth A-|-B--C-|-D--E, and giveth a Rule to reſolve the ſame.

The eigth and laſt ſuppoſeth the whole Rank, firſt, Affirmative, and the ſecond evenly or alternately leſs: And giveth a Solution thereof.

A further Explanation of this Table will be when we come to calculate the Area of any part of the Curvilined Trapezia, A⋜ΘYC.

The Second Table.

I	II	III	IV
Complement Ordinates ſtanding upon CD (more than Radius) or Perpendiculars let fall from	Ordinates or Perpendiculars ſtanding upon DB prolonged.	Arithmetical Complements.	What Proportion the Second Colum hath to Radius.
10	10	00	1
20	5	5	$\frac{1}{2}$
30	333333333	666666666	$\frac{1}{3}$
40	2,5	7,5	$\frac{1}{4}$
50	2	8,	$\frac{1}{5}$
60	1,66666666	8,33333333	$\frac{1}{6}$
70	1,42857143	8,57142857	$\frac{1}{7}$
80	1,25	8,75	$\frac{1}{8}$
90	1,11111111	8,88888888	$\frac{1}{9}$
10	1.	9.	$\frac{1}{10}$

This Table conſiſteth of four Columns, The firſt is equal ſpaces numbred in the ſide DB prolonged, or the Tangents greater than Radius.

The

The ſecond ſheweth the length of the Perpendiculars ſtanding upon the ſide DB prolonged; which are Tangents leſs than Radius, and by the Tops paſs the Curve or Hyperbolical line.

The third Column is the Arithmetical Complements.

The fourth Column ſheweth what Proportion the ſecond Column hath to Radius.

The Rectangle or Parallelogram of the firſt and ſecond Column is equal always to the ſquare AD.

This Table is of uſe to find Points to deſcribe the Curve or Hyperbolical line, or to examine if the Curve paſs through ſuch points as the Table mentions.

The making of this Table hath been formerly ſhewn, when it was taught how to deſcribe the Curve.

We now come to ſhew how to make a Table to find the length of the Baſes of the Compound Curvilined Trapezias, or Hyperbolical Spaces.

We call that a Compound Curvilined Trapezia or Hyperbolical Space, when AC is in the Middle of that Baſe.

So AC ſtanding upon the Middle of ΠX hath Perpendiculars or Sides Π≥ and XV, ſo is the Curvilined Trapezia Π≥AVX, to be hereafter underſtood a Compound Curvilined Trapezia or Hyperbolical Space, and will be ſhewn as followeth to be equal to the aforeſaid Spaces CAVEN, and to AVEZB for the Logarithm Space of 2.

And the Compound Curvilined Trapezia △⊙AVEN will be equal to the Curvilined Trapezia AVFOC, and to AVF⚵B for the Logarithm Space of 3.

The Compound Curvilined Trapezia or Hyperbolical Space Π≥AVX we may prove to be equal to AVEZB thus, by the 43 of the firſt of *Euclid* the Parallelogram CK is equal to K 5, and the Curvilined Triangle AVK common to both, ſo then is AVKXC equal to AV5B. And the Parallelogram Π ⸸ equal to the Square VZ, and the Curvilined Triangle ≥ ⸸ A equal to the Curvilined Triangle V♒E, and ſo the Compound Curvilined Trapezia Π≥AVX equal to the Curvilined Trapezia AVEZB for the Logarithm Space of 2. For by the *4th* Table following, look what Proportion the Perpendiculars or Sides of the Compound Curvilined Trapezias have one to the other, the like Proportion have the Sides or Perpendiculars of the other two Curvilined Trapezias.

So in this Compound Curvilined Trapezia ΠΣ and XV the Sides or Perpendiculars are in a Proportion as 2 is to 1 defcending, or as 1 is to 2 afcending; fo likewife in the Curvilined Trapezia CAVEN (equal to the aforefaid Compound Curvilined Trapezia ΠΣAVX) the Side or Perpendicular NE is double to CA. And alfo in the Curvilined Trapezia AVEZB (equal as before to the Compound Curvilined Trapezia ΠΣAVX) the Side or Perpendicular BA twice ZE as before taught.

Thus by the Ratio of the 2 Tables following may you make a Compound Curvilined Trapezia equal to either of the other two Curvilined Trapezias or Hyperbolical Spaces, and the calculating the Area of the Compound Curvilined Trapezias will be found to be of far greater Difpatch than the former Method, by which we fhall make ufe of *Dairy*'s Rule, or rather the learned *James Gregory*'s from his firft Inference on his 5 Propofition.

We come now to infert the third Table, which is a Table of Ratios to find the Length of the Bafes of the Compound Curvilined Trapezias.

You may note, that in all the three different forts of Curvilined Trapezias or Hyperbolical Spaces equal the one to the other, if on the Middle of their Bafes, you fhall erect Perpendiculars to touch the Curve the greater part or fegments in each is equal to either greater fegment of the other, and fo is the leffer part or fegment of the one equal to the leffer fegment of either of the other.

The

The Third Table.

Being a Table of Ratios to find the Length of the Bafes of the Compound Curvilined Trapezias or Hyperbolical Spaces.

I		II		III		IV	V	VI
5D	:	DC	::	DC	:	DX		
15	:	10	::	10	:	,666666666	For 2	Length of the Bafe 666666666
5D	:	ZD	::	DC	:	DΠ		
15	:	20	::	10	:	1, 333333333		
ZD	:	DC	::	DC	:	DN		
20	:	10	::	10	:	5	3	10
ZD	:	ΦD	::	DC	:	DΔ		
20	:	30	::	10	:	15		
25	:	10	::	10	:	4	4	12
25	:	40	::	10	:	16		
30	:	10	::	10	:	333333333	5	1333333333
30	:	50	::	10	:	1 666666666		
35	:	10	::	10	:	285714285	6	1428571430
35	:	60	::	10	:	1. 714285715		

This Table confifteth of fix Columns, The firft four fhew the Proportion or Ratio to find the Lengths of the Bafes, and the Number in the fixth Column is the Length of the Bafe for fo many Spaces as the fifth Column fignifies.

And by the fame Reafon you may find the Lengths of the Bafes for any other Curvilined Trapezia or Hyperbolical Space.

Thus is *666666666* of the fixth Column (the difference of the two firft Numbers in the fourth Column) the Length of the Bafe, for the Curvilined compound Trapezia or Hyperbolical Space to reprefent the Logarithm of 2.

And 10 the Length of the Bafe for 3, fo is 12 for 4: and 1,333333333 for 5, and fo is 1428571430 for the Length of the Bafe for the Hyperbolical Space for *6*. And thus may you do for any other Space or Number.

The Numbers in the fourth Column for 2, 3, 4, 5, *6*, *&c.* are in Proportion as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, *&c.* And added are equal to twice Radius or 20 = D☿.

We proceed next to fhew how to make a Table of Ratios to find the Lengths of both the Perpendiculars (or fides) of the Compound Curvilined Trapezias.

The

The Fourth Table.

Being a Table of Ratios to find the Length of both the Perpendiculars or Sides of the Compoun Curvilinedd Trapezias or Hyperbolical Spaces·

I		II	III	IV	V
CD— CX	= XD	: DB	:: DB	: D5 = XV	
10 — 333333333	= 666666666	: 10	:: 10	: 15	2
CD + CΠ (= CX)	= DΠ	: DB	:: DB	: ΠΣ	
10 + 333333333	= 1333333333	: 10	:: 10	: 7, 5	
CD—CN	= ND	: DB	:: DB	: DZ = NE	
10 — 5	= 5	: 10	:: 10	: 20	3
CD + CΔ (= CN)	= DΠ	: DB	:: DB	: Δ⊙	
10 + 5	= 15	: 10	:: 10	: 666666666	
CD—C♋	= ♋D	: DB	:: DB	: DR = ♋♊	
10 — 6	= 4	: 10	:: 10	: 25	4
CD + C♉ (= D♋)	= D♉	: DB	:: DB	: ♈♉	
10 + 6	= 16	: 10	:: 10	: 625	
CD—C♌	= ♌D	: DB	:: DB	: DΦ = F♌	
10 — 666666666	= 333333333	: 10	:: 10	: 30	5
CD + C♍ (= C♌)	= D♍	: DB	:: DB	: ♍♎	
10 + 666666666	= 1666666666	: 10	:: 10	: 6	

35 583333333	6
40 571428571	7
45 5625	8
50 555555555	9
55 55	10

This Table confifteth of five Columns, The firft contains the Quantities and Numbers of the firft Term in the Proportion; The fecond Column the Quantities and Numbers of the fecond Term in the proportion; The third Column the Quantities and Numbers of the third Term in the Proportion; The fourth Column the Quantities and Numbers of the fourth Proportional Number or Term, wherein are Numbers for the Length of both the Perpendiculars for 2, 3, 4, 5, *&c.* The fifth Column is the Numerical Order of the Compound Curvilined Trapezias or Hyperbolical Spaces of 2, 3, 4, 5, *&c.*

And

And by the ſame Ratio you may find the Lengths of both the Perpendiculars for any other Compound Curvilined Trapezias to repreſent the Logarithm of any other Number.

By the fourth Column you may perceive the Perpendiculars or Sides (of the Compound Curvilined Trapezias or Hyperbolical Spaces) are in ſuch Proportion the one to the other, as the Number they repreſent are to Unity.

That is to ſay. In the Compound Curvilined Trapezia- ΠΣAVX to repreſent the Logarithm Space of 2. the Perpendi ular XV is to ΠΣ as 2 is to 1.

And in the Compound Curvilined Trapezia △⊙AVEN to repreſent the Logarithm Space of 3, the Perpendicular NE is in proportion to △⊙ as 3 is to 1.

And ſo the Perpendiculars of the fourth Logarithm Space as 4 to 1. And of the fifth Space as 5 to 1, *&c.* as by the fourth Column of this fourth Table appeareth.

And the Perpendiculars of both the other ſorts of the Curvilined Trapezia or Hyperbolical Spaces are likewiſe in the very ſame Proportion the one to the other, as you may note from what hath been ſaid before of them.

By theſe Tables and by what hath been ſaid formerly; theſe three Curvilined Trapezias have the ſame Properties and Affections as thoſe have in an Hyperbola derived from the Section of a Right Angled Cone.

We ſhall now therefore come to calculate ſome part of this latter Hpperbolical Space before we ſhew, how to do it all at once; that is of the Hperbolical Space △⊙AVEN to calculate the Area of the Space △⊙AC which is equal as before ſhewn to the Space of 5VAB. And when we have ſhewn to calculate this part, we ſhall from this, and what hath been taught how to do the other Part come to derive *Dairy*'s Rule, or rather *James Gregory*'s, which is compriſed in the firſt Inference on his 5*th* Propoſition.

CHAP. VII.

WE have in the 5*th*. Chapter calculated the Area of the Curvilined Trapezia or Hyperbolical Space CAVEN equal to AVEZB for the Logarithm of 2.

In

In this Curvilined Trapezia CAVEN all the Perpendiculars ſtanding upon the Baſe CN are each more than Radius CA (or greater than the Tangent of 45*d.* 00′.) being ſtill aſcending and affirmative, and thefore by the 1*ſt.* Table to be continually ad- as by the Calculation thereof is alſo manifeſt.

We now come to calculate the Curvilined Trapezia C△⊙A part of the Curvilined Trapezia C♈⊙⊙A equal to AVEZB.

In this Curvilined Trapezia C△⊙A (equal to B5VA) all the Perpendiculars ſtanding upon the Baſe CA are each leſſer than the Radius CA being ſtill deſcending and negative, and therefore to be handled by the firſt Table accordingly.

The Baſe C△ is equal to CN of that Space calculated as before in the 5*th*. Chapter.

If by the Vertex A you draw a Parallel to the Diagonal CB as ZA♈ it is a Tangent to the Curve touching it in the point A, and AB doth cut all the Perpendiculars contrarywiſe to CB. For CX=X♍ is not equal to ♈Π=Π♉ but Π♉ is equal to K♍=KB becauſe CΠ is equal to CX, and the Angle Π♈♉ equal to the Angle KB♍ ſo therefore by the 1 and 4 Table all the Perpendiculars ſtanding upon C♈ are leſſer than Radius. And ſeeing by the ſixth Column of the firſt Table, and alſo by the 4*th.* of the fourth Table we may find the Length of △⊙. Therefore to know the Area of C△⊙A making C⊕ the firſt Term of an Infinite Series in a continual Proportion, as CA is to C△ that is as 50 to 25 what 25? *facit* 125, as in the Infinite Series of Numbers continually Proportional for the Calculation of the Logarithms of 2 in Chap. 5. you do therefore as there ſaid from 50 (of the ſecond part under the Title, the Quotes to be added) ſubſtract 125, and to that add 416666666666666, and from that ſubſtract 15625 and ſo on throughout, you ſhall have 405465108108165 for the Area of the Space C△⊙A equal to BAV5. And thus may you find any other part of C♈ΠA.

We ſhall ſhew how to do it for CΠΣA and CXVA, becauſe from them we ſhall derive *Dairy*'s Rule or rather *James Gregory*'s, for from them we have derived and calculated the Logarithm for 2 to twenty five places, as by the Calculation following next after this will appear.

Now

Now to Calculate the Area of the Curvilined Trapezia or Hyperbolical Spaces CΠΓA and CXVA, you make C or CH the First Term of any Infinite Series, and the Second Term in such a Ratio as ΠC is to CA for your Proportionals of your Infinite Series, and so proceed on as in *Chap*. V. and as here appeareth.

The Infinite Series of Numbers Proportional.					*The Quotes to be added for* CXVA.		
a	333333333333333.	I	A	+	333333333333333	+	A
aa.	111111111111111.	II	B	—	55555555555555	+	B
aaa	.37037037037037.	III	C	+	12345679012345	+	C
a^4	.12345678012345.	IV	D	—	.3086419753086	+	D
a^5	..411522633744 8.	V	E	+	..823045267489	+	E
a^6	..137174211248 3.	VI	F	—	..228623685414	+	F
a^7	...457247370827.	VII	G	+	...65321052927	+	G
a^8	...152315790275.	VIII	H	—	...19051973784	+	H
a^9	....50805263425.	IX	I	+	5645029269	+	I
a^{10}	....16935087808.	X	K	—	1693508781	+	K
a^{11}	.....5645029269.	XI	L	+	513184479	+	L
a^{12}	.....1881676423.	XII	M	—	156806369	+	M

We have but gone through Twelve Steps of this Calculation, to shew the manner thereof; but should you proceed to go through it till it works off, as in *Chap*. V. you may have both the Segment CXVA, and CΠΓA; for if you finish the Calculation, and add up all the Quotes, that Sum will be the Area of CXVA, and be found 405465108108165, as in *Chap*. V. and is equal to the greater Segment WNHE ☽ in the Curvilined Trapezia or Hyperbolical Space CNHE ☽ A, and also WNHE ☽ is equal to C △ ⊙ A equal to AB 5 V.

And if from 33333333333333 you shall substract the Second Number 55555555555555, and to that add the Third Number 234567901 2345, and then from that substract the Fourth, and so add and substract according to the Signs + and — throughout, you will have .. 287682072451780 for the Area of the lesser Segment of the Compound Curvilined Trapezia WΠΓA ☽ that is the Area of CΠΣA equal to CWDA equal to V 5 ZE.

 And

And you have not only gotten by this Calculation the Area of each Segment ſeparately, and ſo conſequently the Area of the whole Space, by Addition of thoſe Two, but you have alſo obtained the half of the whole Area at once, for if you ſhall (correſpondent to the Column of the Firſt Table) add the Numbers with + affirmative, they will give you half the Area of the compound Curvilined Trapezia XΠΓAV for the Logarithm of 2, which you will ſee preſently exemplified and done to 25 places: And this is the Summ of *James Gregories* Inference on his Fifth Propoſition of his *Exercitationes Geometricæ*; and ſo agreeable to the Rule delivered to me as before declared by *Mich. Dairy*: Having acquainted ſeveral Perſons with *Dairy's Rule* in page 1, and ſhewn to them ſome figurative work thereupon in Order to make a Logarithm, I was notwithſtanding ſome time through inadvertency almoſt diſcouraged of ever knowing how to Cube *in infinitum* ſuch a Number as there ſpoken of, neither did any of thoſe to whom I had communicated the ſame, take any ſuch notice thereof (that I know) ſo as to do it. And now I come to ſhew how I overcame that difficulty of Cubing a Range of Figures for 25 Places, which he told me I muſt do *in Infinitum*, before I could make the Logarithm of ſo many places; and to remove this ſtumbling Block (I do confeſs) took up ſome time; for *Dairy* had not then told me a word of ſuch an Authour as *James Gregory*, and I had not known his Works, but for *John Collins*, ſome years after *Dairy's* Death; but before I ever met with *Gregory's* Book, I had obtained my deſire to Cube *in Infinitum* Twenty Five Figures, That is Twenty five 3 by dividing by 9 continually, as in the Calculation following, to find the Logarithm of 2 all at once, which manner diſpatcheth the Calculation much more ſpeedy than the Method of Calculation in the Fifth *Chap.*

And now the Reaſon of Cubing Twenty Five 3, by dividing only by 9 doth follow.

For as much as *Dairy*'s Rule before declared, to make the Logarithm of 2, doth bid you to 2, add 1, and from 2 ſubſtract 1; ſo ſhall there be a Reſult or Fraction of $\frac{1}{3}$, and then divide 1 or 100, 000, 000, 000, 0000 by $\frac{1}{3}$, whoſe Quotient is 33333333333333, which Cube *in Infinitum*, it had been as much as if he had ſaid Cube $\frac{1}{3}$ fractionally, which is $\frac{1}{27}$, and divide 1000000000000000 by $\frac{1}{27}$ the Quotient will be

be 37037037037037 for the Cube of a or $\frac{1}{3}$, as in the Operation before going. Now for as much as you would Cube the Number for $\frac{1}{3}$, *viz.* 333333333333333 (which is 1, or 100, 000, 000, 000, 0000, divided by 3) it is, as if you should say as 27 to 1000000000000000: what 1? the Quotient will be 37037037037037 for the Cube of a or 333333333333333, as before. Now if you shall, as in the Operation before-going, set down 333333333333333 (which is equal to $\frac{1}{3}$,) you have no more to do but to divide by 9, for that $\frac{1}{9}$ of $\frac{1}{3}$ is equal to $\frac{1}{27}$, and therefore dividing 333333333333333 by 9, the Quotient will be 37037037037037O, as before for the Cube of $\frac{1}{3}$ or a, and seeing $\frac{1}{3} \times \frac{1}{3}$ is equal to $\frac{1}{9}$, you have no more to do but to divide continually by 9, and they shall all be Proportional Numbers by 7th of the 8th of *Euclid*, and consequently correspondent to the odd Powers; for if the Root be multiplied by the Square, that begets the Cube, and the Cube again by the Square, that begets the Fifth Power, and so on. So here, for as much as dividing by 9 doth beget the Third Power; if you shall therefore continually divide by 9, you shall have the respective odd Powers accordingly, as is also manifest by the last Figurative Calculation; and all is, for that a 1 doth neither multiply nor divide, and that $\frac{1}{9}$ of $\frac{1}{3}$ is equal to $\frac{1}{27}$, and if you shall divide $\frac{1}{27}$ of 1, by 9, the Quotient will be 333333333333333, which is equal to a for the First Number or Root, as before.

Now for as much as to make a Compound Curvilined Trapezia equal to an uncompounded: As for instance, to make the Compound Curvilined Trapezia WΠΓΑ ☽ to be equal to the uncompounded CAVEN equal to ABZE = A⊙♈ for the Logarithm of 2, and to find the Length of the Base, and both the Perpendiculars, hath been discoursed, and may be seen, as in the Third and Fourth Table before-going. We come to handle and calculate the Area of this compounded Curvilined Trapezia WΠΓ☽A for to make the half Logarithm of 2 at once.

Seeing by the Sixth Column of the Third Table, the Base WΠ is *6666666666666666*, whose half is . . 333333333333333 for CW or CΠ equal to the First Term in the former Operation (and also the same as *Dairy*'s Result or Fraction of $\frac{1}{3}$.) and that I must divide in the Ratio of AC to CΠ or CW *in In-*

 finitum,

finitum, as in the Fifth Chapter, and alſo as in this is ſhewn and taught, for to make the Infinite Series of Numbers Proportional: It will appear that if I do divide 33333333333333 by 9, it will give me the Cube of the Firſt Term, and ſo dividing continually by 9, will produce the Numbers appertaining to the odd Powers, as by the large Calculation to 25 places next following: And ſeeing I am by *Dairy*'s Rule or rather *James Gregory*'s, to divide each of the Numbers of the Infinite Series by the Indices of the odd Powers, it is manifeſt, That this Rule of *Dairy*'s is derivable from the 8 Column of the Firſt Table, for $A + B + C + D + E + F + G + H + I$

And $A - B + C - D + E - F + G - H + I$

doth make $^2A + {}^2C + {}^2E + {}^2G + {}^2I + {}^2L + {}^2N$:

And therefore every other Line of the Quotes to be added in the former Operation, doth make half the Logarithm of 2.

In making the Infinite Series in page 34, in Order to make the half Logarithm of 2, to 25 places be very careful to ſet the figures in their due places, and to make that Series you are to divide continually by 9, which being done throughout, you may then prove your work by *Multiplieation* in multiplying each line by 9, and if thoſe Multiplications produce the foregoing Numbers you may conclude that part of the work to be well prepared. And ſeeing by the direction over the figurative Work in page 35, you are to divide each of the Numbers in page 34, by 1, 3, 5, 7, 9, *&c.* You muſt ſo order the Quotes of page 35, that they may lye in the ſame line or range with their reſpective Dividends or Numbers in Page 34: for the better Preventing miſtakes, the letter Figures do repreſent the Diviſors proper to each line; and would you make the Logarithm of 2, according to that Method in page 34 and 35, for 7 or 8, places only, you may very well produce it in half an hours time as by that Calculation is very perceptible. And ſome that have had thoſe two ſheets I formerly Printed as a Specimen hereof, have told me they have done the ſame, and were very ſolicitous I would as ſoon as I could, publiſh the remainder, which at length as time and leiſure hath permitted is done: and though I have not here inſerted many Examples; yet by what are herein done you may perceive how to proceed for any other Number propoſed. And with the direction and reference in page 46, thoſe that are willing and curious herein may make a Logarithm for

any

any natural Number desired. I have not added hereto any Table of Logarithms at this time, and what I may do hereafter in order thereunto I do not presume to promise. I doubt not but some may both examime some Table or other, or make by this Method one *De Novo*, and satisfie themselves about the same, and some have told me since my communicating this Method unto them, that if the first makers of Tables of these Numbers had made them by such easie ways, they did not doubt but their Tables might have been somewhat more exact. Howsoever it pleased God who is the giver of every good and perfect Gift, to raise and endue such men with great ability and patience to perform those Tables with so much difficulty and labor as their Methods did require, and for common Uses sufficient. And with such Eagerness did that Age embrace and pursue the Invention of these Numbers that *Vllach* a Dutchman had exhibited a Table of Logarithms to 10 places for 100000 before the Learned *Henry Brigg*'s Table, which he had in part done to 15 places, could be accomplished by him. So exceeding glad were they of the Invention. And the Learned *Henry Briggs* in his Epistle Dedicatory to Our Most Gracious King's Father when Prince of *Wales*, saith, that amongst the Antients there is not found any Footsteps of these Numbers; of whose Construction and Uses the said *Henry Briggs* hath written in his *Arithmetica Logarithmica* most learnedly and copiously, and now follows the figurative part of making the Logarithm of 2 to 25 Places.

The Infinite Series or Numbers continually Proportional. These Numbers are continually divided by 9, in order to make the Half Logarithm of 2.

a	3333333333333333333333333	I
a	370370370370370370370370	III
a^5	41152263374485596707819	V
a^7	4572473708276177411980	VII
a^9	508052634252908601331	IX
a^{11}	56450292694767622370	XI
a^{13}	6272254743863069152	XIII
a^{15}	696917193762563239	XV
a^{17}	77435243751395915	XVII
a^{19}	8603915972377324	XIX
a^{21}	955990663597480	XXI
	106221184844164	XXIII
	11802353871574	XXV
	1311372652397	XXVII
	145708072489	XXIX
	16189785832	XXXI
	1798865092	XXXIII
	199873899	XXXV
	22208211	XXXVII
	2467579	XXXIX
	274175	XLI
	30464	XLIII
	3385	XLV
	376	XLVII
	42	XLIX

Differentia 1
—
Unitas 1
Numerus Proposit. 2
Summa — 3

$= \overset{a}{\frac{1}{3}} \times \frac{1}{3} = \overset{aaa}{\frac{1}{9}}$

You may perceive that if 1 be added to 2, and substracted from it, it leaveth a Result of $\frac{1}{3}$, which multiplied into it self, maketh $\frac{1}{9}$, and therefore these Numbers are continually divided by 9.

These

These Numbers are Quotes from those on the opposite Side, by dividing them by 1, 3, 5, 7, 9, &c. and are $\frac{{}^2A+}{2}$ $\frac{{}^2C+}{2}$ $\frac{{}^2E+}{2}$ $\frac{{}^2G+}{2}$ $\frac{{}^2I+}{2}$ $\frac{{}^2L+}{2}$ $\frac{{}^2N+}{2}$, &c. Correspondent to the last preceding Calculation, which added make half the Area of the Compound Curvilined Trapezia XΠΓAV for the Half Logarithm of 2 to 25 Places.

3333333333333333333333333	A
1234567901234567901234567	C
823045267489711934156 4	E
6532105297537396302 83	G
56450292694767622370	I
5131844790433420216	L
482481134143313012	N
46461146250837549	P
4555014338317407	R
452837682756702	T
45523364933213	W
4618312384529	Y
472094154863	
48569357496	
5024416293	
522251156	
54511063	
5710683	
600222	
63271	
6687	
708⁵	
75	
8	

Half the Logarithm of 2. { 346573590279972654708616 0⁵

The Logarith. of 2. { 693147180559945309417232 1

Thus

Thus have we Calculated the Logarithm for 2 to 25 Places, after *Dairy*'s Rule, or rather *James Gregory*'s, which Method maketh far greater diſpatch than that in *Chap.* V. for this Calculation though to 25 Places, is ſooner performed than that of 15 Places in *Chap.* V. as by comparing them is very perſpicuous and manifeſt.

And now we have exemplified the Rule *Dairy* declar'd, and I am apt to believe he had ſtudied well *Gregory*'s ſaid *Exercitationes*, though he was not pleaſed to tell any more thereof, but that others ſhould take pains therein as well as he, and truly if *John Collins* had not acquainted me with *Gregory*'s Works, I had done the Work, but not with that ſatisfaction I met with from *James Gregory*'s Books ; and here you have it in a more familiar Diſcourſe and Dialect than that of *James Gregory*'s, being altogether Analitical, and if any Letter or Symbol be miſtaken in his, it is very great Study and Labour to find, and to ſet it to rights.

I find *James Gregory* hath calculated the Hyperbolical Logarithm for 2 in his *Vera Circuli & Hyperbolæ Quadratura* to 25 Places, which agreeth with this Calculation, but to 17 Places I have not raiſed the Logarithm for 2 to his Doctrine in that Book, but am ſatisfied this Calculation for the Logarithm of 2 in this Chapter is true to an Unite in the 25th Place, and may be in Two hours very well Examined by any one that will take the pains to do it, and they ſhall find it to be as herein Calculated. And to this I have the Concurrence of the moſt ingenuous and laborious Mr. *Abraham Sharp*, who (from the Occaſion of my publiſhing formerly two ſheets of the Praxis hereof as a Specimen) hath ſhewn me his Calculation of the Logarithm of 2, and ſome others to forty Places, the like I ſuppoſe not hitherto heard of or ſeen. Without all doubt *Gregory* found that *Mercators* Lucky Invention of Squaring the Hyperbola, was of far more diſpatch than that of his *Vera Circuli & Hyperbolæ Quadratura*, or elſe he would not have Writ upon *Mercator* : But ſo excellently hath *Gregory* Illuſtrated *Mercator*, that a better way of Squaring the *Hyperbola* I ſuppoſe hath not nor may be found.

We

We ſhall for the future keep to this laſt Method for Calculating thoſe Examples hereafter following. And now we proceed to calculate a Logarithm for 1 $\frac{1}{4}$, for having got the Logarithm of 2, we have the Logarithm of 4, and all the Powers thereof; ſo then if we make a Logarithmof 1 $\frac{1}{4}$, and add to it the Logarithm of 4, we ſhall have the Logarithm of 5, becauſe that 4 X 1 $\frac{1}{4}$ makes 5; that is, 4 multiplyed by 1 $\frac{1}{4}$, makes 5: And this Method I took to make the Logarithm of 5, and having got the Logarithm of 5, to the Logarithm of 5 add the Logarithm of 2, and you have the Logarithm of 10. And when I have ſhewn this, I ſhall produce *Brigg*'s Logarithm of 2, by one ſingle Diviſion; for that all ſorts of Logarithms are Proportional the one to the other; we therefore now haſten to make the Logarithm for 1 $\frac{1}{4}$.

The Calculation of the Logarithm for 1 $\frac{1}{4}$ to 25 Places.

The Infinite Series or Numbers continually Proportional; theſe Numbers are continually divided by 81.

		I
a	1111111111111111111111111	III
a^3	13717421124828532235940	V
a^5	169350878084302867111	VII
a^7	2090751581287689718	IX
	25811747917131972	XI
	318663554532493	XIII
	3934117957191	XV
	48569357496	XVII
	599621697	XIX
	7402737	XXI
	91392	XXIII
	1128	
	14	

Theſe Numbers are Quotes from th on the oppoſite Side, they being di ded by 1, 3, 5, 7, 9, &c. *which ded, make half the Logarithm* 1 $\frac{1}{4}$ *to* 25 *Places.*

1111111111111111111111111
4572473708276177411980
33870175616860573422
298678797326812817
2867971990792441
28969414048408
3026244582245
323795716б
3527186
38961
435
4

111571775657104877883147
This Numb. is $\frac{1}{2}$ the Log. for 1
223143551314209755766295
This Number is the Log. of 1

Adding 1 to 1 $\frac{1}{4}$, it maketh 2 $\frac{1}{4}$, and ſubſtracted from it, leaveth $\frac{1}{4}$, which Reſult maketh a Fraction of $\frac{1}{9}$, for 2 $\frac{1}{4}$ being reduced into Fourths, make $\frac{9}{4}$, ſo the Reſult of $\frac{1}{4}$ and $\frac{9}{4}$ (rejecting the Denominators) is $\frac{1}{9}$ as above, which Squared maketh $\frac{1}{81}$; ſo are theſe Numbers therefore continually divided by 81, to make the Infinite Series. By Decimals it preſently ſheweth it ſelf to be a Fraction of $\frac{1}{9}$. Thus the Difference or Numerator is 25 the Summ or Denominator 2,25, Which Decimal Fraction $\frac{25}{2,25}$ is equal to $\frac{1}{9}$.

Differentia	25
Unitas	1
Numerus Propoſitus	1,25
Summa	2,25

To divide the Numbers on the other ſide (to make the Infinite Series) by 81, is eaſie enough, for it is but dividing twice by 9, or taking one Ninth part twice, and rejecting or cancelling the firſt, ſo is it very readily done, and the whole Operation hereof may very well be performed in two hours time; and thus have we got the Logarithm for 1 $\frac{1}{4}$ to 25 places, and now ſhall proceed to make the Logarithm of 10, which is by adding together the Logarithm of 2, 3 times, and that makes the Logarithm of 8, and that added to the Log. of 1 $\frac{1}{4}$, makes the Log. of 10; and the Logarithm of 2 ſubſtracted from the Logarithm of 10, leaveth the Logarithm of 5, and is to the ſame effect as is before.

Logarithm	of	2.	6931471805599453094172321
Logarithm	of	8.	20794415416798359282516963
Logarithm	of	1 $\frac{1}{4}$.	2231435513142097557662951
Logarithm	of	10.	23025850929940456840179914
Logarithm	of	5.	16094379124341003746007593

We have now made and exhibited the Logarithms of 2, 5, and 10, and from theſe you may make all their Compoſites.

And now we proceed to make the Logarithm of 3 to 25 Places, which we ſhall ſhew two ways, firſt, all at once from a Compound Curvilined Trapezia or Hyperbolical Space; ſecondly, by a Compoſition of 2 Logarithms, *viz.* of 2 and 1 $\frac{1}{2}$, for that 2 X 1 $\frac{1}{2}$ maketh 3, and this latter we chiefly recommend. The Compound Curvilined Trapezia or Hyperbolical Space N△⊙AVE we have in the foregoing *Chapter* ſhewn to be equal to the uncompounded CAVFO, and alſo equal to AVFΦB,

we

we ſhall Calculate the Logarithm for 3, according to the Compounded Space, and by the Third and Fourth Tables, you may know the Lengths of the Baſe and Perpendiculars.

The Baſe N△ is 10, therefore CA = 5 = CX. Now for as much as *Dairy*'s Rule of adding to, and ſubſtracting 1 from 3, produces the half Length of the Baſe agreeable to the Third Table, we ſhall ſhew how to Calculate the half of the Logarithm for 3, as we did for the half Logarithm of 2.

Adding 1 to 3 it makes 4, and ſubſtracting 1 from 3, leaveth 2, which maketh a Reſult or Fraction of $\frac{2}{4} = \frac{1}{2}$.

Now dividing 1 and 25 Cyphers by $\frac{1}{2}$, the Quotient is 5, for the firſt Term of your Infinite Series Now for as much as $\frac{1}{2} \times \frac{1}{2}$ maketh $\frac{1}{4}$, the ſecond Term muſt be therefore $\frac{1}{4}$ of the Firſt, and ſo on, as was diſcours'd before in making the half Logarithm of 2.

Having made the Infinite Series as followeth, you divide each of thoſe Numbers (which as before Taught are proportional) by 1, 3, 5, 7, 9, *&c.* and theſe added, make half the Area of N△ΦAVE for the half Logarithm of 3.

The

The Infinite Series or Numbers continually Proportional. These Numbers are continually divided by 4, in order to make the ½ Log. of 3 to 25 Places.

a		5000000000000000000000000
aaa		125
$aaaaa$		3125
a^7		78125
a^9		1953125
a^{11}		48828125
a^{13}		1220703125
a^{15}		30517578125
a^{17}		762939453125
a^{19}		19073486328125
a^{21}		476837178203125
		11920928955078125
		298023223876953125
		74505805969238281
		18626451492309570
		4656612873077392
		1164153218269343
		291038304567336
Differentia 2		72759576141834
—		18189894035458
Unitas — 1		4547473508864
Numerus	a aaa	1136868377216
Propositus 3	$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	284217094304
—		71054273576
Summa — 4		17763568394
		4440892099
		1110223025
		277555756
		69333936
		17333485
		4333371
		1083343
		270836
		67709
		16927
		4232
		1058
		264

These

These Numbers are Quotes from those on the opposite side, those being divided by 1, 3, 5, 7, 9, &c. which added make half the Log. of 3 to 25 Places.

I	5000000000000000000000000
III	416666666666666666666666
V	625
VII	11160714295714285714285
IX	2170138888888888888888
XI	443892045454545454545
XIII	93900240384615384615
XV	2034505208333333333
XVII	4487879136029411765
XIX	1003867701480263158
XXI	227065313430059523
XXIII	51830125891644022
XXV	11920928855078125
XXVII	2759474295156975
XXIX	642291430759295
XXXI	150213318486367
XXXIII	32247067220283
XXXV	8315389130495
XXXVII	1966475030861
XXXIX	466407539294
XLI	110913988021
XLIII	26438799470
XLV	6315935429
XLVII	1511793055
XLIX	362521804
LI	87076316
LIII	20947604
LV	5046469
LVII	1216385
LIX	293788
LXI	71039
LXIII	17196
LXV	4167
LXVII	1010
LXIX	245
LXXI	59
LXXIII	15
LXXV	3
Half the Log. of 3.	5493061443340548456976226
The Logarithm of 3.	10986122886681096913952452

We

We ſhall now proceed to make the Logarithm for 3 the ſecond way, which is from the Logarithms of 2 and $\frac{1}{2}$.

The Infinite Series or Numbers continually Proportional. Theſe Numbers are continually multiplied by 4, in order to make the Half of the Logarithm of $1\frac{1}{2}$ to 25 Places.

a	2000000000000000000000000
aaa	8
$aaaaa$	32
a^7	128
a^9	512
a^{11}	2048
	8192
	32768
	131072
	524288
	2097152
	8388608
	33554432
	1342177
	53687
	2147
	85

Differentia $\frac{1}{2}$

Unitas — 1

Numerus Propoſitus $1\frac{1}{2}$ $= \overset{a}{\frac{1}{5}} \times \frac{1}{5} = \overset{aaa}{\frac{1}{25}}$

Summa $2\frac{1}{2}$

To make this Infinite Series, I ſhould divide by 25 continually, but if you multiply by 4, and transfer it anſwerable, it will be the ſame thing: Becauſe $\frac{1}{5}$ of 10 is $\frac{2}{1}$, and that multiplied in it ſelf; is $\frac{4}{100}$. Therefore multiplying by 4 and dividing by 100, is the ſame thing as multiplying by 25; And thus this Infinite Series is made very ſpeedily, in order to make the half of the Logarithm for 1 and $\frac{1}{2}$.

Theſe

These Numbers are Quotes from those on the opposite side, they being divided by 1, 3, 5, 7, 9, 11, &c. *and added, make half the Logarithm for* 1 $\frac{1}{2}$ *to* 25 *Places.*

I	2000000000000000000000000
III	266666666666666666666666
V	64
VII	182857142857142857I4
IX	5688888888888888888
XI	186181818181818I8
XIII	630153846153846
XV	21845333333334
XVII	7710117647 06
XIX	27594105204
XXI	998643809
XXIII	36472209
XXV	1342177
XXVII	49711
XXIX	1851
XXXI	69
XXXIII	3

Half the Logar. of 1 $\frac{1}{2}$.	2027325540540821909890065
The Logarithm of 1 $\frac{1}{2}$.	4054651081081643819780131
The Logarithm for 2.	6931471805599453094172321
The Logarithm for 3.	10986122886681096913952452

Having now made the Logarithm for 1 $\frac{1}{2}$, you add to it the Logarithm of 2, and that makes the Logarithm for 3, which will be found as before to be the same Number.

And now we proceed to make the Logarithm for 7, and then we shall have all to 11. In order thereunto, we make the Logarithm for 1, $\frac{2}{5}$ or 1,,4, and add that to the Logarithm of 5, and it will produce the Logarithm of 7, for that 1 $\frac{2}{5}$ multiply by 5, maketh 7, or 1,4 X 5 = 7.

The

The Infinite Series or Numbers continually Proportional: These Numbers are continually divided by 36, *in order to make half the Logarithm of* 1 $\frac{2}{5}$ *or* 1 ,4.

a	16666666666666666666666
·	2777777777777777777777
aaa	462962962962962962962 96
·	77160493827160493827 16
aaaaa	12860082304526748971 19
·	2143347050754458161 86
a^7	357224508459076360 31
·	59537418076512726 72
a^9	9922903012752121 12
·	1653817168792020 19
a	275636194798670 03
	45939365799778 64
	7656560966629 72
	127609349443 829
	21268224907304
	3544704151217
	590784025203
	98464004200
	16410667366
	2735111228
	455851871
	75975312
	12662552
	2110425
	351737
	58623
	9737
	1623
	270

Differentia 4

Unitas 1

Numerus Propositus 1 4 $= \overset{a}{\frac{1}{6}} \times \frac{1}{5} = \overset{aaa}{\frac{1}{36}}$

Summa —— 2 4

&c.

This Series is made by dividing Twice by 6, which is all one as if you divided at once by 36, and so every other Number is the proper Number of the Series to be divided by 1, 3, 5, 7, 9, 11, *&c.* as in the other side to make the half Logarithm for 1 ,4.

☞ The Logarithm of 5 in page 38 being put in the room of that in page 45 will produce 19459101490553133051053 53 for the true Logarithm of 7. Those two last Numbers in page 45 being part mistaken.

These

These Numbers are Quotes from those on the opposite side, those being divided by 1.3. 5. 7. 9. 11, &c. And added make half the Logarithm of $1\frac{2}{5}$ or $1\frac{4}{10}$.

	. . . .
I	1666666666666666666666666
III	15432098765432098765432
V	257201646090534979424
VII	5103207263701090863
IX	110254477819468014
XI	2505783589078819
XIII	58896622820229
XV	1417881660487
XVII	34752001483
XIX	863719335
XXI	21707237
XXIII	550545
XXV	14069
XXVII	360
XXIX	9
Half the Logarithm of $1\frac{4}{10}$	1682361183106064652522972
The Logarithm of $1\frac{2}{5}$ or $1\frac{4}{10}$	3364722366212129305045944
The Logarithm of 5.	16094379124341003168895254
The Logarithm of 7.	19459101490553132473941198

Having by this Calculation made the half Logarithm of 1 $\frac{4}{10}$. if we double it, and to that add the Logarithm of 5 that Addition will produce the Logarithm of 7 as was required. And now we have all the Logarithms to 11, and to make the Logarithms from 10 to 100, it will not be much difficult to proceed after the foregoing Methods, as to make the Logarithm of 11, you have for the first Term *a*, the result or fraction $\frac{5}{6}$, and for *a a a*, it will be $\frac{25}{36}$, which is very easie to work. And for the Logarithm of 13, you make it of 12 multiplyed by 1 $\frac{1}{12}$. And so it is for the first Term *a*, the result or fraction $\frac{1}{25}$, and for the second *a a a*, it is $\frac{1}{625}$, which 625 is = to $\frac{5}{8}$, and so may you make many easie compendiums for the Prime Numbers between 10 and 100, and also not with great difficulty from 1000 to 10000, and when you have made some Logarithms you will perceive how the differences arise, and having for Composites, Logarithms in a readiness, greater and lesser than the Prime or Incomposite very near, it will be by the Difference no great difficulty to make a Logarithm for such a Prime very readily and easily. And they that are curious herein may have Compendiums hereof in *James Gregory*'s aforesaid *vera Circuli & Hyperbolæ Quadratura* to make Logarithms for Prime or Incomposite Numbers, to which I shall refer him; and here I shall content my self to have exemplified *James Gregory*'s Method in his *Exercitationes Geometricæ* to so many Examples of Logarithms as I have herein Calculated to 25 Places, and shall in the next Chapter shew how to produce from these Geometrical Logarithms *Brigg*'s Logarithms.

CHAP.

CHAP. VIII.

HAving in the Preceeding Chapter made the Logarithms for 2, 3, 4, 5, 6, 7, 8, 9, and 10, according to the Geometrical Figure or Hyperbola, I require the Logarithm of 2 according to *Brigg*'s Table. For as much as all Logarithms are Proportional, it is as the Hyperbolical Logarithm of 10, is to its Logarithm of 2 :: So is *Brigg*'s Logarithm of 10 to his Logarithm of 2. The Operation followeth,

This Divifor is half the Logarithm of 10, according to the Hyperbola.

Diviſor

1 1 5 1 2 9 2 5 4 6 4 9 7 0 2 2 8 1

Quotient.

3 0 1 0 2 9 9 9 5 6 6 3 9 8 1 1 9 0

This Quotient is the Logarithm of 2, according to *Briggs* his Table.

By this Diviſion it doth appear, that this Quotient doth agree with *Briggs* his Table of Logarithms for his Logarithm Number of 2, whereby it is apparent he did produce the Logarithm for 2 to 15 places very true, though I have been told it was eight Perſons work for a years time after his Method, which was by large and many Extractions of the Square Root, and if it was ſo to 15 places, it would have been very tedious (if not impoſſible) for them to have produced the Logarithm of 2 to 25 places, as before herein is ſhewn and done by us, and both the Hyperbolical and *Brigg*'s Logarithm to 25 places may very well be calculated and done according to the foregoing Method in half a days time, by which Method herein before going one may make a Table of Logarithms in a ſhort Space to what *Pardie* in his Elements of Geometry (a French Author) hath declared, for he ſaith, he knew more than 20 perſons engaged for 20 years with indefatigable aſſiduity to calculate the Logarithms. He doth not ſay to how many places: But the greateſt Radius that I have ſeen of any French Author is but 11 places, which I ſuppoſe muſt be but the ſame as *Vulack*'s. And the Logarithm for 2, 3, 4, or 5, *&c.* to 11 places according to the Method in this Book may be very well done and performed in leſs than two hours time.

This Dividend is compounded of half the Hyperbolical Logarithm of 2, and *Briggs* his Logarithm of 10.

Dividend

Dividend

. . . .

346573590279997264, 00000000000000 000

1185820330865797 .

345337843687741 9 .

115079334388337338

114630052036052851

110137228513207981

65208993284759281

76443659599081405

73561068092600364

45835153027789954

112963766328792697

93474371440605431

137096772084 36262

219675174346603 39

104445919096901109

82959051216 90561

The Reader may now fee that Logarithms derived from this Figure or the Hyperbola are not only more perceptible and intelligible, but with far more Certainty and Expedition produced than what was known in former times.

The Divifor in the foregoing work differs 2 Unites in the 18place, from the half Logarithm of 10 before herein calculated, and the Reafon is, that I took *Gregory*'s Logarithm of 10 in his *Vera Circuli & Hyperbolæ Quadratura de bene effe*, and having calculated the half Logarithm of 2 as before, I was very defirous to fee if we could produce *Brigg*'s Logarithm of 2 to 15 places, as by the Divifion is manifeft, and this I did, long before I met with *Gregory*'s other Book of his *Exercitationes Geometricæ* ; for fince I got that Book I did calculate *De Novo* the Logarithm of 10 to 25 places according to his Doctrin in that Book, and as before herein is done. And the Calculation of the Logarithm of 10 as before doth agree with *Gregory*'s former Book but to 17 places ; howfoever the Divifion before going is fufficient to produce *Brigg*'s Logarithm for 2 to 15 places, and if any fhall be fo curious to produce *Brigg*'s Logarithm for 25 places,

places, he may rely on the foregoing Examples herein, and may in 4 hours time examine the foregoing Calculations thereof, and in as little time produce *Brigg*'s Logarithm for them to the like Number of places.

Having this Diviſion ready done, long before the publiſhing hereof, I have contented my ſelf to inſert it here, whereby the ſtudious may ſoon perceive what to do further to gratify himſelf herein.

I do not add hereto any Table of Logarithms, that being not my deſign at this time, but only to ſhew how *Brigg*'s or any other Logarithms may be derived from the Doctrin before going, and alſo for the curious at his will and pleaſure to examine whether any Logarithms formerly publiſhed be truly made or not.

As for the various Uſes of Logarithms I add none here, but refer the Reader to ſuch Authors (whereof there is plenty,) who have long before written largely and learnedly, as the firſt Inventor the famous Lord *Neper*, *Henry Briggs*, *Edm. Gunter*, *Rich. Norwood*, *Wingate*, and divers others; as alſo my Father *John Speidell*, in which the Reader may meet with many excellent Uſes of the Logarithms in all parts of the Mathematicks; and I do find my Father printed ſeveral ſorts of Logarithms, but at laſt concluded that the Decimal or *Brigg*'s Logarithms were the beſt ſort for a ſtandard Logarithm, and did alſo print the ſame ſeveral ways, ſo ordered, whereby they might be applyed to Arithmetical Queſtions and other Operations for the Solution thereof with eaſe and readineſs.

F I N I S.

E R R A T A.

PAge 9. Line 5, read 7 *and* 8 *of Euclid.*
Page 31. Line 25. for $\frac{1}{7}\frac{1}{2}$ read $\frac{1}{2}\frac{1}{7}$.
Page 38 Line 10. for 2 25. read $\frac{1}{2},\frac{2}{2}\frac{5}{5}$.
Page 39. Line 5. for 2 + $1\frac{1}{2}$. read 2 × $1\frac{1}{2}$.